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Heat Transfer over an Unsteady Stretching Surface with Variable Heat Flux In The Presence Of Aheat Source or Sink by Fuzzy Adomian Decomposition Method

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Abstract: *Unsteady boundary layer flow of an incompressible fluid over a stretching surface in the presence of a heat source or sink is studied. The unsteadiness in the flow and temperature fields is caused by the time dependence of the stretching velocity and the surface heat flux. The nonlinear boundary layer equations are transformed to nonlinear ordinary differential equations containing the Prandt l number, heat source or sink parameter and unsteadiness parameter. These equationsare solved by applying Fuzzy Adomian technique and compared with the existing numerical results obtained by using Shooting with Runge Kutta method. This focuses on solving the nonlinear ordinary and partial differential equations using Fuzzy Adomian decomposition method.*

Keywords: *Fuzzy Adomian Decomposition Method, non-linear ordinary differential equation ,non-linear partial differential equation, Runge Kutta Method.*

I. Introduction

The study of two-dimensional boundary layer flow due to a stretching surface is important in variety of engineering applications such as cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of paper and plastic sheets. In all these cases, a study of flow field and heat transfer can be of significant importance since the quality of the final product depends on skin friction coefficient and surface heat transfer rate.

Heating and cooling of fluids finds many industrial applications in power transmittion, manufacturing and electronics. Effective cooling techniques are greatly needed for cooling todays high-energy devices. Conventional heat transfer fluids such as water, ethylene glycol, and engine oil have poor heat transfer capabilities due to their low heat transfer properties. Further, as the thermal conducticities of metals are nearly three times higher than these fluids, it would be desirable to combine the two substances to produce a heat transfer medium that behaves like a fluid with the thermal conductivity of a metal.

The problem of heat transfer from boundary layer flow driven by a continuous moving surface is of importance in a number of industrial manufacturing processes. Several authors have been analysed in various aspects of the pioneering work of Sakiadis (1961). Crane (1970) have investigated the steady boundary layer flow due to stretching with linear velocity.Vleggaar et al. (1977) have analysed the stretching problem with constant surface temperature and Soundalgekar et al. (1980) have analysed the constant surface velocity.

Hashim et al. (2006) applied Adomian decomposition method to the classical Blasius equation. Wazwaz (1997) used Adomian decomposition method to solve the boundary layer equation of viscous flow due to a moving sheet . Awang Kechil and Hashim (2009) used Adomian decomposition method to get the approximate analytical solution of an unsteady boundary layer problem over an impulsively stretching sheet. The heat transfer over an unsteady stretching surface with prescribed heat flux discussed in detail by Ishak et al. (2003).

Awang Kechil and Hashim (2009) applied Adomian decomposition method to a two by two system of nonlinear ordinary differential equations of free-convective boundary layer equation. Hayat et al. (2009) analysed the MHD flow over a nonlinearly stretching sheet by employing the Modified Adomian decomposition method.

II. Formulation Of The Problem

The formulation of the problem presented by Elsayed M.A. Elbashbeshy et al.(2010) is described below
Consider the unsteady two-dimensional laminar boundary layer flow of an incompressible fluid over a continuous moving stretching surface. Assume that the surface is stretched with velocity$U\omega(x,t) = \frac{\alpha x}{1-\gamma t}$ along

the *x* axis by keeping the origin fixed, where the *y* axis is normal to the *x* axis and also assume that the surface being subjected to a variable heat flux $Q\ \omega(x,t) = \frac{\beta x}{1-\gamma t}$.

The basic boundary layer equations that govern momentum and energyrespectively are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0 \tag{4.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \tag{4.2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty) \tag{4.3}$$

subject to the boundary conditions

$$y = 0 : u = U_\omega,\ v = 0,\ \frac{\partial T}{\partial y} = -\frac{q_\omega}{k} \tag{4.4}$$
$$y \to \infty: u = 0, T = T_\infty$$

where *u* and *v* are the velocity components in the the *x* and *y* directions respectively, *T* is the fluid temperature inside the boundary layer, *t* is the time, *k* is the thermal conductivity, *v* is the kinematics viscosity, c_p is the specific heat at constant pressure, ρ is the density, Q>0 represents a heat source and Q <0 represents a heat sink, T_∞ is the temperature far away from the stretching surface, and α,b and γ are,constants,where $\alpha > 0, \beta \geq 0$, $\gamma \geq 0$ and $\gamma t < 1$. Both $\alpha\ and\ \gamma$ have dimension $(time)^{-1}$.

The equation of continuity is satisfied if we choose a stream function ψ(x, y) such that $u = \frac{\partial\varphi}{\partial y}$, $v = \frac{-\partial\varphi}{\partial x}$.

The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables:

$$\eta = \sqrt{\frac{\alpha}{v(1-\gamma t)}}\,y$$
$$\psi(x,y) = \sqrt{\frac{\alpha v x^2}{(1-\gamma t)}}\,f(\eta) \tag{4.5}$$
$$T = T_\infty + \frac{q_\omega}{k}\left[\sqrt{\frac{v(1-\gamma t)}{\alpha}}\right]\theta(\eta).$$

Substituting (4.5) into (4.2) and (4.3), we obtain the following set of ordinary differential equations:

$$f''' + ff'' - f'^2 - A\left[f' + \frac{1}{2}\eta f''\right] = 0 \tag{4.6}$$
$$\theta'' + Pr\left[f\theta' - f'\theta - \frac{A}{2}(\theta + \eta\theta') + \delta\theta\right] = 0 \tag{4.7}$$

The boundary conditions (4.4) now become

$$\eta = 0:\ f = 0, f' = 1, \theta' = -1$$
$$\eta \to \infty: f' = 0\ , \theta = 0 \tag{4.8}$$

where the primes denote differentiation with respect to $\eta, A = \frac{\gamma}{\alpha}$ is a parameter that measures the unsteadiness,Pr = $\frac{\mu c_\rho}{k}$ is the Prandtl number (μ is the viscosity), $\delta = \frac{Q_k}{\mu c_\rho}\frac{Re_x}{Re^2{}_k}$ is the dimensionless heat source or sink parameter, $Re_x = \frac{U_\omega}{v}x$ is the local Reynolds number, and $Re_k = \frac{U_\omega\sqrt{k}}{v}$. The physical quantities of interest in this problem are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as

$$C_f = \frac{\mu\left[\frac{\partial u}{\partial y}\right]_{y=0}}{[\rho U^2{}_\omega/2]}\ ,\ NU_x = \frac{-x\left[\frac{\partial T}{\partial y}\right]_{y=0}}{T_\omega - T_\infty}$$

III. Adomian Decomposition Method

To solve the system of coupled ODEs using Adomian decomposition method, rearranging (4.6) and (4.7) as follows

$$f''' = -ff'' + f'^2 + A\left[f' + \frac{1}{2}\eta f''\right] \quad (4.9)$$

$$\theta'' = -Pr\left[f\theta' - f'\theta - \frac{A}{2}(\theta + \eta\theta' + \delta\theta\right] \quad (4.10)$$

by applying the standard procedure of Adomian decomposition method
Eqs (4.9) and (4.10) becomes

$$L_1 f = \left[-ff'' + f'^2 + A\left[f' + \frac{1}{2}\eta f''\right]\right] \quad (4.11)$$

$$L_2\theta = -PrL\left[\left[f\theta' - f'\theta - \frac{A}{2}(\theta + \eta\theta') + \delta\theta\right]\right] \quad (4.12)$$

Where

$L_1 = \frac{d^3}{d\eta^3}$ and inverse operator $L^{-1}{}_1(.) = \int_0^\eta\int_0^\eta\int_0^\eta(.)d\eta d\eta d\eta$ and

$L_2 = \frac{d^2}{d\eta^2}$ and inverse operator $L^{-1}{}_2(.) = \int_0^\eta\int_0^\eta(.)d\eta d\eta$

Applying the inverse operator on both sides of (4.11) and (4.12)

$$L^{-1}{}_1 L_1 f = L^{-1}{}_1\left[-ff'' + f'^2 + A\left[f' + \frac{1}{2}\eta f''\right]\right] \quad (4.13)$$

$$L^{-1}{}_2 L_2\theta = -PrL^{-1}{}_2\left[\left[f\theta' - f'\theta - \frac{A}{2}(\theta + \eta\theta') + \delta\theta\right]\right] \quad (4.14)$$

Simplify eqs (4.13) and (4.14) we get

$$f(\eta) = \eta + \frac{a\eta^2}{2} + \int_0^\eta\int_0^\eta\int_0^\eta\left[-N_1(f) + N_2(f) + A\left[f' + \frac{1}{2}\eta f''\right]\right]d\eta d\eta d\eta \quad (4.15)$$

And

$$\theta(\eta) = b - \eta - Pr\int_0^\eta\int_0^\eta\left[N_3(f,\theta) - N_4(f,\theta) - \frac{A}{2}(\theta + \eta\theta') + \delta\theta\right]d\eta d\eta \quad (4.16)$$

Where $a = f''(0) and\ b = \theta(0)$ are to be determined from the boundary conditions at infinity in (8). The non linear terms $ff'', f'^2, f\theta' and\ f'\theta$ can be decomposed as Adomian polynomials $\sum_{n=0}^{\infty} B_n, \sum_{n=0}^{\infty} C_n, \sum_{n=0}^{\infty} D_n and\ \sum_{n=0}^{\infty} E_n$ as follows

$$N_1(f) = \sum_{n=0}^{\infty} B_n = ff'' \quad (4.17)$$

$$N_2(f) = \sum_{n=0}^{\infty} C_n = (f')^2 \quad (4.18)$$

$$N_3(f,\theta) = \sum_{n=0}^{\infty} D_n = f\theta' \quad (4.19)$$

$$N_4(f,\theta) = \sum_{n=0}^{\infty} E_n = f'\theta \quad (4.20)$$

Where

$B_n(f_0, f_1, \ldots\ldots, f_n), C_n(f_0, f_1, \ldots\ldots, f_n)\ and\ D_n(f_0, f_1, \ldots\ldots, f_n, \theta_0, \theta_1, \ldots., \theta_n), E_n(f_0, f_1, \ldots\ldots, f_n, \theta_0, \theta_1, \ldots., \theta_n)$ are the so called Adomian polunomials. In the Adomian decomposition method (1986) f and θ can be expanded as the infinite series

$$f(n) = \sum_{n=0}^{\infty} f_n = f_0 + f_1 + \cdots + f_m + \cdots$$

$$\theta(\eta) = \sum_{n=0}^{\infty}\theta_n = \theta_0 + \theta_1 + \cdots + \theta_m + \cdots \quad (4.21)$$

The individual terms of the Adomian series solution of the equation (4.6)-(4.8) are provided below by the simple recursive algorithm

$$f_0(\eta) = \eta + \frac{a\eta_2}{2} \quad (4.22)$$

$$\theta_0(\eta) = b - \eta \quad (4.23)$$

$$f_{n+1}(\eta) = \int_0^\eta\int_0^\eta\int_0^\eta\left[-B_n + C_n + A\left[f_n{}' + \frac{1}{2}\eta f''\right]\right]d\eta d\eta d\eta \quad (4.24)$$

$\theta_{n+1}(\eta) = -Pr\int_0^\eta \int_0^\eta \left[D_n - E_n - \frac{A}{2}(\theta_n + \eta\theta_n{'}) + \delta\theta\right] d\eta d\eta$ (4.25)

For practical numerical computation , we take the m-term approximation of $f(\eta) and\ \theta(n)$ as

$\phi_m(\eta) = \sum_{n=0}^{m-1} f_n(\eta)$and

$$\omega_m(\eta) = \sum_{n=0}^{m-1} \theta_n(\eta)$$

IV. Results Analysis

The recursive algorithms (4.22)–(4.25) are programmed in MATLAB. We have obtained upto 15th term of approximations to both $f(\eta) and\ \theta(n)$. The first few terms are given as follows:

$$f_0(\eta) = \eta + \frac{a\eta^2}{2}$$

$$f_1 = \left[\frac{A}{6} + \frac{1}{6}\right]\eta^3 + \left[\frac{Aa}{16} + \frac{a}{24}\right]\eta^4 + \left[\frac{a^2}{120}\right]\eta^5$$

$$f_2 = \left[\frac{-a^3}{40320}\right]\eta^8 + \left[\frac{a^2A}{1120} - \frac{a^2}{5040}\right]\eta^7 + \left[\frac{aA^2}{192} + \frac{aA}{240} + \frac{a}{720}\right]\eta^6 + \left[\frac{A^2}{60} + \frac{A}{60}\right]\eta^5$$

And

$$\theta_0(\eta) = b - \eta$$

$$\theta_1 = \mathrm{Pr}\left[\left[\frac{2b + Ab - 2b\delta}{4}\right]\eta^2 + \left[\frac{\delta - A + ab}{6}\right]\eta^3 - \left[\frac{a}{24}\right]\eta^4\right]$$

$$\theta_2 = Pr\left[\frac{b}{24} + \frac{Ab}{24} - \frac{Prb}{24} + \frac{APrb}{24} + \frac{A^2Prb}{32} + \frac{Prb\delta^2}{24} - \frac{APrb\delta}{12}\right]\eta^4 - Pr\left[\frac{A}{60} + \frac{Pr\delta}{60} - \frac{ab}{120} + \frac{A^2Pr}{60} + \frac{Pr\delta^2}{120} - \frac{APr}{60} - \frac{APr\delta}{40} - \frac{Aab}{80} + \frac{Prab}{60} - \frac{APrab}{60} + \frac{Prab\delta}{120} + \frac{1}{60}\right]\eta^5 - Pr\left[\frac{-a}{240} + \frac{Aa}{160} - \frac{Pra}{240} - \frac{a^2b}{720} + \frac{APra}{1440} + \frac{Pra\delta}{720} + \frac{Pra^2b}{360}\right]\eta^6 + \left[\frac{Pra^2}{1008} - \frac{a^2}{1260}\right]\eta^7$$

The undetermined values of ***a*** and ***b*** are computed using the boundary conditions at infinity in (4.8). The difficulty at infinity is tackled by applying the diagonal Padé approximants Boyd (1997). that approximate $f'(\eta) and\ \theta(\eta) using\ \phi_{15}{'}(\eta) and\ \omega_{15}(\eta)$. Applying infinity to the diagonal Padé approximants [N/N] that approximates $f'(\eta) and\ \theta(\eta)$ ranging value of N from 2 to 10 provides a two by two system of non linear algebraic equation, then obtained nonlinear system are solved by employing Newton Raphson method. The numerical results of **a** and **b** obtained are shown in the following Tables.

Table 4.1: Comparison of local nusselt number at A=0 and δ = 0 for various values of Pr obtained using ADM and FADM with previously published results.

Pr	Present Result		Elbashbeshy et al. (2010)	Ishak et al. (2008)	Exact solution (1965)
	ADM- Padé of [6/6]	FADM- Padé of [6/6]			
0.72	0.808	0.8081	0.808	0.8086	0.8086
1	1	1	1	1	1
10	3.9921	3.7205	3.7207	3.7202	3.7206

Table 4.2: Comparison of skin friction coefficient and local nusselt number for Pr=1 and δ = -2 at different values of A obtained using ADM and FADM .

A	**Present Result**				**Elbashbeshy et al. (2010)**	
	f "(0)		1 / □ (0)		*f* "(0)	1/□(0)
	ADM-Padé of [6/6]	FADM Padé of [6/6]	ADM-Padé of [6/6]	FADM Padé of [6/6]		
0	1.0108	1.0057	1.786	1.7851	1	1.7844
0.8	1.3227	1.3219	1.8544	1.8541	1.3218	1.854
1.2	1.4528	1.4533	1.8906	1.8904	1.4535	1.8904
2	1.6838	1.6828	1.9676	1.964	1.6828	1.9635

Table 4.3: Comparison of skin friction coefficient and local nusselt number for Pr=1 and δ = 0 at different values of A obtained using ADM and FADM with previously published results.

A	**Present Result**				**Elbashbeshy et.al. (2010)**	
	f "(0)		1/□(0)		*f* "(0)	1/□(0)
	ADM-Padé of [6/6]	FADM-Padé of [6/6]	ADM-Padé of [6/6]	FADM-Padé of [6/6]		
0	1	1	1.0056	1	1	1
0.8	1.3219	1.3218	1.1352	1.1362	1.3218	1.136
1.2	1.4536	1.4535	1.2078	1.207	1.4535	1.207
2	1.6831	1.6828	1.3355	1.3345	1.6828	1.3345

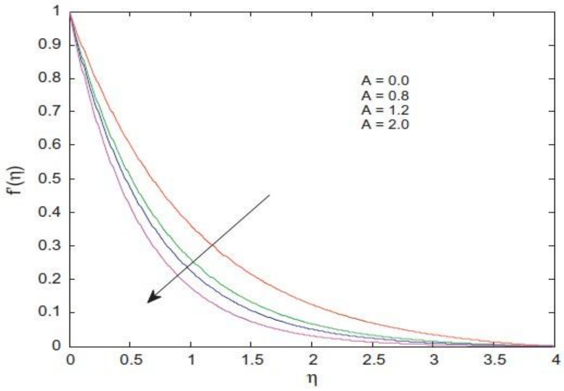

Fig. 4.1 Velocity profiles $f^{'}(\eta)$ for various values of A when Pr= 1 and δ = −2 Using $\varphi'_{15[10/10]}$.

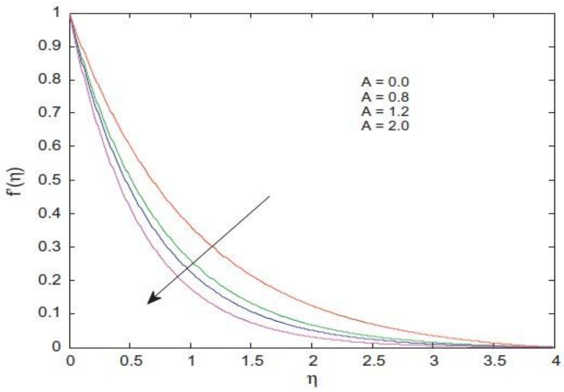

Fig. 4.2 Velocity profiles $f^{'}(\eta)$ for various values of A at Pr= 1, δ = 0.1 Using $\varphi'_{15[10/10]}$.

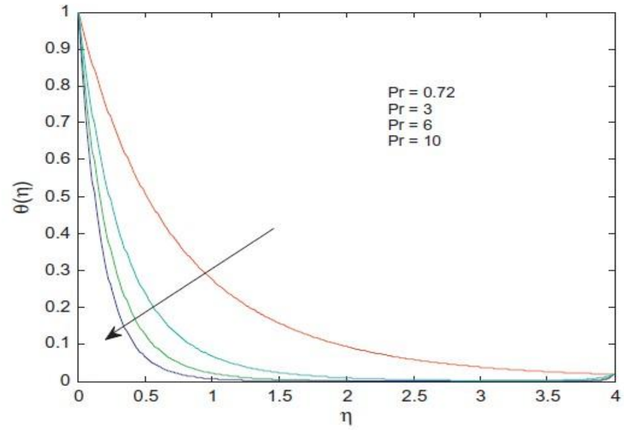

Fig. 4.3 Temperature profiles $\theta(0)$ for various values of Pr at A = 1.2 and δ = -1 Using $\omega_{15[{}^{10}/_{10}]}$.

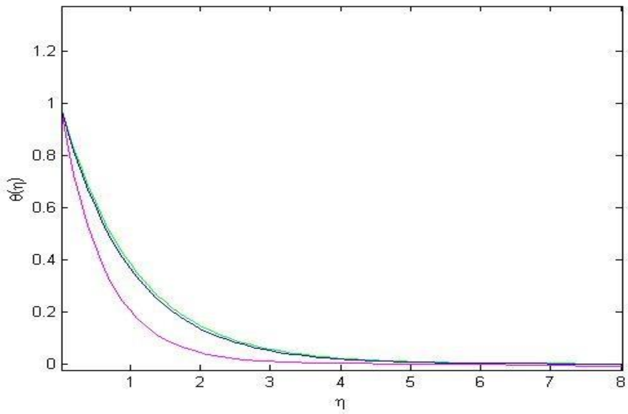

Fig.4.4 Temperature Profiles for various values of Pr at A=0.8 and δ = -0.1 using $\square_{15[8/8]}$.

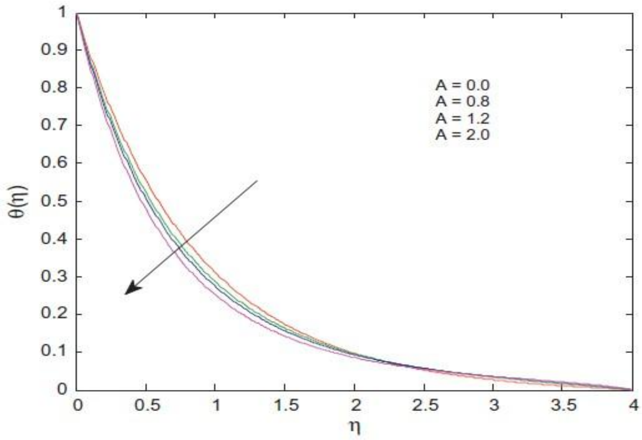

Fig. 4.5 Temperature profiles $\theta(0)$ for various values of A at Pr= 1 and δ = 0.1 using $\square_{15[8/8]}$.

From Figs.4.1 and 4.2, we note that when unsteadiness parameter A increases,the velocity profiles decreases.This implies that the skin friction coefficient increases.In Figs. 4.3 and 4.3 we note that when Prandtl Number (Pr) increases that implies the temperature decreases within the boundary layer for all values of the Prandtl number.This is consistent with the well-known fact that the thermal boundary layer thickness decreases with increasing Prandtl number. In Fig 4.5 we note that when unsteadiness parameter A increases the temperature Profiles is decreases.

V. Conclusion

The Adomian decomposition method and Modified Adomian decomposition method is applied to solve a system of two nonlinear ordinary differential equations with the specified boundary conditions that describes Heat transfer over an unsteady stretching surface with variable heat flux in the presence of a heat source or sink. The obtained solutions have matched with the existing numerical result.The Adomian decomposition method and Modified Adomian decomposition method techniques are very efficient alternative tools to solve nonlinear models with infinite boundary conditions.

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